FTD-TT-61-57/1+2

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UNEDITED ROUGH DRAFT TRANSLATION

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English Pages: 13

SOURCE: Izvestiya Akademii Nauk SSSR - Otdel. Tekh. Nauk (Mekhanika i Mashinostroyeniye), No. 2, March-April 1961, pp. 70-77

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WP-AFB, OHIO.

FTD-TT-61-57/1+2

Date 30 Nov. 1961

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Equations are dervied which express stationary and nonstationary flows of rarefied gas around bodies under conditions of free-molecular flow, where it is necessary to take into consideration the repeated reboundings of gas particles from the surfaces of the bodies around which the flow is directed. Due to reasons, which are explained in this report, these equations are formulated without imposition of any kind of limitations on the law of reflection. In the appendices there is described a method of determining the reflective properties of a surface, possessing a certain finely granular structure; an important special case of the reflection law is discussed, representing a combination of diffusional and mirror reflection; a simplified nonstationary instance is examined where nonstationary movements which are superimposed over the stationary ones, appear to be small; and finally we discuss the flow around bodies of a stream, the parameters of which execute small harmonic oscillations around their mean values.

If in the flow of a rarefied gas several bodies are simultaneously situated arranged so, that their reciprocal influence is noticeable, or even one body, the surface of which has depressions, it is necessary to take into consideration the repeated reflections of gas particles from various parts of the surface of bodies around which the flow is directed. This problem is discussed here for a combination of bodies, the dimensions of which are such, that the flow flowing around it can be considered as free-molecular. This requires that the characteristic length,

accepted in the problem, should be smaller than the smallest of the lengths of the free path in the gas for particles of the oncoming flow and for reflected particles. When this condition is maintained it can not only be considered that the particles of the oncoming flow have an unperturbed velocity distribution, but also that between the subsequent reflections from the surface the particles are not scattered.

At such a simplified arrangement, it would appear, that it is possible directly to find the distribution function for the gas. We disregard the reaction of the particles, and the distribution function should satisfy the Louville equation and depends consequently, upon time and the invariants of free particle $f = f(v, [rv], t)$. The form of function f should be determined from the boundary and initial conditions. This latter operation appears to be difficult, and here is suggested another approach to the solution of the problem, connected with the derivation of an integral equation for the density of the flow of particles reflected from the surface around which the flow is directed, which in the given case replaces the kinetic equation.

Applying this method in report [1] we investigated the flow around bodies with absolute diffusional reflectring surface. As is shown by experiment, polished surfaces can with a certain accuracy be treated as absolute diffusion reflecting, and for smooth bodies it would not be necessary to introduce into the investigation more complex reflection laws. It might, however, appear to be convenient to produce surfaces with a certain proper finely-grained structure, because in this way it is possible to change of their streamline considerably. This structure can be considered on the average, by introducing a suitable reflection law, which can be determined from experimental or theoretical considerations, as it will be shown later on. Next are derived equations for stationary flow around at any arbitrary reflection law, namely with this latter case in mind. It is only natural, that during diffusion as well as at any other kind of reflection law the density of the impacts can be determined

independently from the temperature distribution. Such a possibility disappears during nonstationary flow around, when the density of the impacts (collisions) and temperature distribution should be determined parallel. The nonstationary flow equations proposed in this report are obtained in a quite simple manner, by generalizing stationary flow equations.

1. Stationary flow. Flow around surfaces, which alike diffusional-scattering reflect so that the distribution of particles of the reflected stream by angles depends only upon the point on the body, can be described, after finding only the number of particles colliding against unit of surface per unit of time. In the presence of a relationship between incident flow and reflected one the latter should be described in greater detail, including the angular velocity distribution of the particles. For this we will introduce into the investigation functions $n_i(P, \omega)$, $n_r(P, \omega)$, so that the products $n_i d\sigma_p d\omega$ and $n_r d\sigma_p d\omega$ will designate the number of particles falling (reflected) on the element of surface near point P, the velocity of which has the trend included in the solid angle $d\omega$ with axis ω.

To determine function n_r it is possible to obtain an integral equation, which appears to be the result of the condition of preserving the number of particles during the reflection. The form of this equation depends upon the law of reflection of the surface of the body around which the flow has been directed. Assuming that $R(P, \omega_i, \omega_r) d\omega_r$ - probability that the particle falling on the surface near point P with a velocity having the trend ω_i, acquires upon reflection a velocity with trend in solid angle $d\omega_r$. Next it is important that R does not depend (or depends only slightly) upon the absolute velocity value of the incident particle and upon the temperature of the surface. The missions of function R are sufficient to describe the stationary flow, because it can be expected, that at the discussed here low densities in the process of reflection the principle of superposition will be

maintained. The preservation of the number of particles during reflection is expressed by the fact that

$$\int R(P, \omega_i, \omega_r)\, d\omega_r = 1 \qquad (1.1)$$

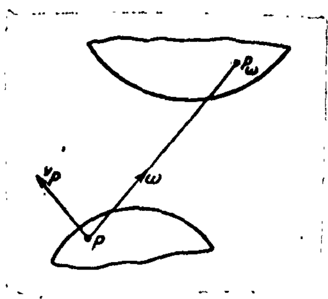

Fig.1.

The relationship between n_r and n_i is written in form of

$$n_r(P, \omega_r) = \int R(P, \omega, \omega_r)\, n_i(P, \omega)\, d\omega \qquad (1.2)$$

In order to transform this relation into an equation to determine n_r we like to point out, thaat the particles moving in direction ω arrive either from the oncoming stream, or from that section of the surface near point P where the direction ω intersects the body for the first time (fig.1). It is apparent that

$$n_i = (P, \omega) = n_i^*(P, \omega) + \frac{r^2_{PP_\omega}}{\omega\nu_{P_\omega}} \frac{\omega\nu_P}{r^2_{PP_\omega}}\, n_r(P_\omega, \omega) \qquad (1.3)$$

where n_i - density of collisions of particles of the unpesturbed incident flow, r_{PQ} - distance between points P and Q, and ν_p - single vector of the normal in point P. The scalar products are always in absolute value.

If this expression for n_i is substitued in formula (1.2) then the latter acquires the form of

$$n_r(P, \omega_r) = \int R(P, \omega, \omega_r)\left[n_i^*(P, \omega) + \frac{\omega\nu_P}{\omega\nu_{P_\omega}}\, n_r(P_\omega, \omega)\right] d\omega \qquad (1.4)$$

This is the sought for equation for n_r - basic equation of the problem. It can be formed differently, changing integration by angles with integral by surface. For this we make a change

$$P_\omega \rightarrow Q, \qquad d\omega \rightarrow \frac{\omega_{PQ}\nu_Q}{r^2_{PQ}}\, d\sigma_Q, \qquad \omega \rightarrow \omega_{PQ} \qquad (1.4a)$$

and we will obtain instead (1.4)

$$n_r(P, \omega) = n_r^*(P, \omega) + \int_{S_P} R(P, \omega_{PQ}, \omega)\, \frac{\omega_{PQ}\nu_P}{r^2_{PQ}}\, n_r(Q, \omega_{PQ})\, d\sigma_Q \qquad (1.5)$$

Here

$$n_r^*(P, \omega) = \int_{S_P} R(P, \omega_i, \omega)\, n_i^*(P, \omega_i)\, d\omega_i\,. \qquad (1.6)$$

and the integration is done over that part of the surface S_p which is visible from point P.

We will show that this equation has a single solution. We will take for this purpose a uniform equation corresponding to equation (1.5)

$$n_r(P, \omega) = \int\limits_{S_P} R(P, \omega_{PQ}, \omega) \frac{\omega_{PQ} \nu_P}{r_{PQ}^2} n_r(Q, \omega_{PQ}) \, d\sigma_Q \qquad (1.7)$$

and will convince ourselves, that it has no other solutions except the trivial $n_r = 0$. Having integrated this equation according to ω and taking into consideration (1.1) we will obtain

$$\int n_r(P, \omega) \, d\omega = \int\limits_{S_P} \frac{\omega_{PQ} \nu_P}{r_{PQ}^2} n_r(Q, \omega_{PQ}) \, d\sigma_Q \qquad (1.8)$$

We will again integrate over the entire surface of streamlined bodies

$$\int d\sigma_P \int n_r^*(P, \omega) \, d\omega = \int d\sigma_P \int\limits_{S_P} \frac{\omega_{PQ} \nu_P}{r_{PQ}^2} n_r(Q, \omega_{PQ}) \, d\sigma_Q \qquad (1.9)$$

In this equality the left part characterizes the general number of particles, reflected by the body per unit of time, and the right part the number of particles which is reflected from the body and falling on it again. In the case of flowing around as is discussed here (but not for movements in closed zone) the first number should alwqys be greater than the second one, in view of the fact that a part of the reflected particles departs into infinity. Formula (1.7) consequently, cannot be fulfilled.

2. Unsteady Flow. We will formulate an equation of nonstationary flow in the case, when only the parameters of the outer flow are changed, and the geometric parameters remain unchanged. Generally speaking, to write an equation for an unstationary flow is needed a more detailed description of the properties of the reflecting surface, which was necessary in the stationary case. But the basic difficulty, which emerges here, lies in the fact, that the density of the collisions can no longer be determined independently from temperature distribution; values n_r at the moment t appear to be dependent upon the temperature values in all foregoing moments.

This leads to the point where the equation of the problem in essence becomes non-linear. The equations to determine the temperature will be written out here; it will be considered as a fixed function of position and time.

The reflection process is considered as occurring instantaneously; we deal in disregarding not of time the incident particles remain in the interior of the body around which the flow is directed, which, of course, is very small, but the time of their delay in these macroscopic details of the surface ,about which mention was made in the introduction.

We like to point out, first of all, that the necessity of introducing changes into stationary flow equations originates only at sufficiently rapidly changing state of the system. Namely the parameters of the incident flow or the neometric parameters of the bodies should change noticeably at the time of relaxation, which is of the order here of the average time the particles remain within the limits of the system around which the flow is directed ($\sim 1/v$, where 1 - mean distance covered by the particle between two subsequent reflections, v -certain mean velocity of reflected particles). Otherwise the movement is quasistationary and it is possible to apply equation (1.5), replacing, of course, all the parameters by values at the moment t:

$$n_r(P, \omega, t) = n_r^*(P, \omega, t) + \int\limits_{S_P(t)} R(P, \omega_{PQ}, \omega) \frac{\omega_{PQ} \nu_P(t)}{r_{PQ}^2(t)} n_r(Q, \omega_{PQ}, t)\, d\sigma_Q \quad (2.1)$$

This equation should be solved in the assumption that t - parameter. Here we can have a change not only in form of function n_r, but the very surface around which the flow is directed may become quite deformed.

Let us discuss the particular form of surfaces, which are of special practical importance and the flow around which can be described with the aid of equations (1.3), (1.5). This includes these surfaces which allow to give a description of the stream reflected from them with the aid of the temperature aspect. It means that the distribution of velocity values of particles reflected from any given small

section of the surface is given for all angles by the expression of the form of const $v^2 \exp(-mv^2/2kT_g)$. The temperature T_g included herein generally does not coincide with the temperature of the surface, and is connected with same by the relation of accommodation.

Assuming $d\sigma_P$ and $d\sigma'_Q$ - two such surface elements, where one is visible from the other; let us see how their mutual influence spreads. The particles which arrive from the element $d\sigma_P$ fall on $d\sigma_Q$ with delay r_{PQ}/v. It is apparent, if $T_g(Q,t)$ - temperature of particles leaving the element $d\sigma_Q$ then from their number the following amount will fall on element $d\sigma_P$ during the time interval dt:

See Page 7a for Equation (2.1a)

Hence we obtain a generalization of equation (1.5) for the nonstationary case:

See Page 7a for Equation (2.2)

In spite of all this, this equation was formulated on the basis of a certain schematization, it is still quite cumbersome. A substantial simplification of same can be attain in the following manner. By virtue of the presence of an exponential multiple the time integral in (2.2) is practically taken only in the interval

$$0.5 r_{PQ}\left(\frac{m}{4kT_g}\right)^{1/2} \lesssim \tau \lesssim 1.5 r_{PQ}\left(\frac{m}{4kT_g}\right)^{1/2} \quad (2.2a)$$

The temperature T_g, which is connected with the temperature of the surface, changes, generally speaking, slowly, so that we can write

$$T_g(Q, t-\tau) \sim T_g(Q, t) \quad (2.2b)$$

Making substitution

$$\tau = r_{PQ}\left[\frac{m}{2kT_g(t)}\right]^{1/2} u = \tau_{PQ}(t)\, u \quad (2.2c)$$

we obtain instead of (2.2)

$$n_r(P, \omega, t) = n_r^*(P, \omega, t) + \quad (2.3)$$

$$+\left(\frac{16}{\pi}\right)^{1/2}\int_{S_P} R(P, \omega_{PQ}, \omega)\,\frac{\omega_{PQ} v_P}{r_{PQ}^2}\int_0^\infty u^{-4}\exp(-u^{-2})\, n_r[Q, \omega_{PQ}, t-\tau_{PQ}(t)\, u]\, d\sigma_Q du$$

$$dt\int_0^\infty n_r(Q,\ \omega_{PQ},\ t-\tau)\left(\frac{16}{\pi}\right)^{1/2}\left[\frac{m}{2kT_g\,(Q,t-\tau)}\right]^{3/2}\left(\frac{r_{PQ}}{\tau}\right)^3 \times \qquad (2.1a)$$

$$\times \exp\left[-\left(\frac{r_{PQ}}{\tau}\right)^2\frac{m}{2kT_g\,(Q,\ t-\tau)}\right]\frac{d\tau}{\tau}\,\frac{\omega_{PQ}\nu_P}{r_{PQ}^2}\,d\sigma_P d\sigma_Q$$

$$n_r(P,\ \omega,\ t) \doteq n_r^*(P,\ \omega,\ t) + \int_{S_P} R\,(P,\ \omega_{PQ},\ \omega)\,\frac{\omega_{PQ}\nu_P}{r_{PQ}^2}\int_0^\infty\left(\frac{16}{\pi}\right)^{1/2}\left[\frac{m}{2kT_g\,(Q,\ t-\tau)}\right]^{3/2}\times$$

$$\times\left(\frac{r_{PQ}}{\tau}\right)^3\exp\left[-\left(\frac{r_{PQ}}{\tau}\right)^2\frac{m}{2kT_g\,(Q,\ t-\tau)}\right]n_r\,(Q,\ \omega_{RQ},\ t-\tau)\,\frac{d\tau}{\tau}\,d\sigma_Q \qquad (2.2)$$

The physical idea of the approximation made here is clear: the temperature of body surface should not change sharply during the elapse of time between subsequent reflections; below equation (2.2) is considered in somewhat greater detail.

Application of the temperature concept to the reflected stream, even though it appeared up until now satisfactory for the recording of the energy balance and quantity of motion at the surface, may appear to be highly approximate for the recording of nonstationary flow equations, because these equations are more sensitive to details of the distribution function. The published experiments cannot be used as basis for the evaluation, equation (2.2) has to be presented in a more general form. We will very briefly describe, for reference purposes, how this should be handled. If the distribution of reflected particles by energy value depends upon the angles the properties of the reflecting surface should be given through function

$$R[P, T(P), \omega_i, v_r, \omega_r, v_r] \quad (2.3a)$$

so that

$$R[P, T(P), \omega_i, v_i, \omega_r, v_r]\, v_r^2\, dv_r\, d\omega_r \quad (2.3b)$$

is the probability of the fact that the particle falling with a velocity v_i will appear after reflection in the dv_r element of the velocity space. Consequently the densty of collisions shouls also be broken down into pomponents in velocity space: $n_r(P, \omega_r, v_r) v_r^2 dv_r d\omega_r$ - number of reflected particles with a velocity in the lement dv_r. In result the generalized equation (1.5) acquires the form

$$n_r(P, \omega, v) = n_r^*(P, \omega, v) + \\ + \int_0^\infty v_i^2 dv_i \int_{S_P} \frac{\omega_{PQ} \nu_P}{r_{PQ}^2} R[P, T(P), \omega_{PQ}, v_i, \omega, v]\, n_r(Q, \omega_{PQ}, v_i)\, d\sigma_Q \qquad (2.4)$$

The nonstationary flow equation, corresponding to (2.4) is obtained immediately by changing from integrating by velocities into integrating by time. Assuming that $\tau = r_{PQ}/v_i$, we obtain

$$n_r(P, \omega, v, t) = n_r^*(P, \omega, v, t) + \int_{S_P} \frac{\omega_{PQ} \nu_Q}{r_{PQ}^2} \int_0^\infty \left(\frac{r_{PQ}}{\tau}\right)^{3]} \times \\ \times R\left[P, T(P, t), \omega_{PQ}, \frac{r_{PQ}}{\tau}, \omega, v\right] n_r\left(Q, \omega_{PQ}, \frac{r_{PQ}}{\tau}, t - \tau\right) \frac{d\tau}{\tau_i}\, d\sigma_Q \qquad (2.5)$$

Equation (2.2), (2.3), (2.5) were formulated by us in the assumption that the geometry of the streamlined system is constant. It slow changes may, however, be taken into consideration, just as it was done in (2.1).

Practically up to this moment it was believed that in the flow exists only one kind of particles. If there are several kinds or during reflection the particles dissociate or become ionized, then a separate suitable equation should be written for each type. All these equations are not independent, and form a system. Excited energy states of one and the very same particle should not be considered separately since they have a very short life span and influence only the energy balance.

3. Flow of Energy and Aerodynamic Forces. If the reflection energy n_r has been found then the recording of term for energy flows and for the amount of motion presents no difficulty whatsoever. At first we find the collision density

$$n_i(P, \omega, v, t) = n_i^*(P, \omega, v, t) + \int_{S_P} \frac{\omega_{PQ}\nu_P}{r_{PQ}^2} n_r\left(Q, \omega_{PQ}, v, t - \frac{r_{PQ}}{v}\right) d\sigma_Q \quad (3.1)$$

The flow of the number of movement, falling on the body. i.e. the ~~####~~ force acting per unit of its surface is determined by expression

$$f(P, t) = m\int_0^\infty v^3 dv \int [n_i(P, \omega, v, t) - n_r(P, \omega, v, t)]\, \omega d\omega \qquad (3.2)$$

The stream of energy yielded to the body is obtained in form

$$\varepsilon = \frac{m}{2}\int d\omega \int (n_i - n_r)\, v^4 dv \qquad (3.3)$$

Parts of these expressions, corresponding to n_r^*, for combining diffusional and mirror reflection, can be found in the book by Patterson [2] ; for the flow, executing ~~####~~ oscillations, they were established by N.T. Pashchenko in report [3] .

If upon reflection the particles transform into excited state, then the energy consumed for this should, apparently, be deducted from the right part (3.3).

4. Appendix. We will show briefly how to find the probability of reflections $R(\omega_1, \omega_r)$ for a surface, which is not smooth. Separating from the surface any given

element of its structure we should solve for this element equation (1.3) from report [1], having taken $N^*(P)$ for the single flow with trend ω_1. We will obtain, in this way, a term, determining $N_1^*(P)$ as a function of ω_1. Then

$$R_1^*(\omega_i, \omega_r) = \int N_1^*(P)\nu_P\omega_r d\sigma_P \qquad (4.1)$$

where the integral is taken by the very same part of the structural element which is seen from direction ω_r.

For example, for a surface with spherical depressions one can use solution [1], equation (3.5) for a sphere. For a surface with cylindrical grooves (fig.2) one can use solution for a cylinder, which acquires a simple ~~form~~ form

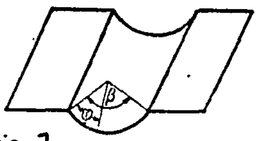
Fig. 2

See page 10a for Equation (4.2)

We shall make a more thorough investigation of a stationary flow in the important special case when the law of reflection correspond to a combination of diffusional and mirror reflections. Equation (1.5) look now

$$n_r(P, \omega) = \gamma\left[n_i^*(P, \omega') + \frac{\omega'\nu_P}{\omega'\nu_{P_{\omega'}}} n_r(P_{\omega'}, \omega')\right] +$$
$$+ (1-\gamma)\frac{\omega\nu_P}{\pi}\left[N^*(P) + \int_{S_P} \frac{\omega_{PQ}\nu_P}{r_{PQ}^2} n_r(Q, \omega_{PQ})\, d\sigma_Q\right] \qquad (4.3)$$

where gamma is the coefficient of proportionality of mirror reflection (specular reflection), $\omega' = 2(\omega\nu_p) - \omega$ - vector, symmetrical ω in ratio to ν_p; a designation was introduced

$$N(P) = \int n(P, \omega)\, d\omega \quad (4.3a)$$

In concrete problems, to which equation (4.3) can be applied, the proportion of the specular reflection os always small. Naturally according to this it is necessary to break down the values figuring in (4.3) according to degrees gamma. We write

$$n_r(P, \omega) = [N(P) + \gamma n_r'(P, \omega)]\,\omega\nu_{P/\pi} \quad n^*(P, \omega) = n'^*(P, \omega)\,\omega\nu_P/\pi \quad (4.3b)$$

In zero approximation we obtain instead of (4.3) and equation

$$N(P) = N^*(P) + \frac{1}{\pi}\int_{S_P} \frac{\omega_{PQ}\nu_P\omega_{PQ}\nu_Q}{r_{PQ}^2} N(Q)\, d\sigma_Q \qquad (4.4)$$

$$N(\varphi) = N^*(\varphi) + \frac{1}{4}\int_0^{\varphi} (\varphi - \varphi_0)\, N^*(\varphi_0)\, d\varphi_0 + A\varphi + B \tag{4.2}$$

$$A = -\frac{1}{8}\int_0^{\beta} N(\varphi_0)\cos\frac{\varphi_0}{2}\, d\varphi_0, \qquad B = \frac{1}{4}\int_0^{\beta} N(\varphi_0)\sin\frac{\varphi_0}{2}\, d\varphi_0$$

which, naturally, coincides with equation (1.3) from report [1], since it describes the flow around an absolute diffusional reflective surface. For hatched values we obtain in first approximation an equation

$$n_r'(P, \omega) = n_i^{*\prime}(P, \omega') + N(P_{\omega'}) - N(P) + \frac{1}{\pi} \int_{S_P} \frac{\omega_{PQ}\nu_P \omega_{PQ}\nu_Q}{r_{PQ}^2} n_r'(Q, \omega_{PQ})\, d\sigma_Q \quad (4.5)$$

This equation is immediately reduced into for of (4.4). To realize such a transformation, we will mention, that function

$$N'(P) = n_r'(P, \omega) - n_i^{*\prime}(P, \omega') + N(P) - N(P_{\omega'}) \quad (4.6)$$

no longer depends upon the angles. For it is obtained an equation corresponding exactly with (4.4), only with

$$\pi^{-1} \int_{S_P} \frac{\omega_{PQ}\nu_P \omega_{PQ}\nu_Q}{r_{PQ}^2} [-n_i^{*\prime}(Q, \omega_{PQ}') + N(Q) - N(Q, \omega_{PQ}')]\, d\sigma_Q \quad (4.6a)$$

instead of N*)P). In this way, the presence of a small fraction of specular reflection does not bring in any substantial difficulties into the solution of problems concerning flow around diffusion reflective surfaces.

We will investigate simply the flow around bodies, which besides stationary motion execute a small nonstationary movement. We investigate this case with the aid of equation (2.3). At first we will make an approximate calculation of the time integral figuring there

$$\sqrt{\frac{16}{\pi}} \int_0^\infty n_r(Q, \omega_{PQ}, t - \tau_{PQ}(t) u)\, u^{-4} \exp(-u^{-2})\, du \sim n_r[Q, \omega_{PQ}, t - 2^{-1/2}\tau_{PQ}(t)] \quad (4.6b)$$

Since the nonstationary movement is considered small, then the shift of the argument can be considered in approximation. Instead of (2.3) is obtained equation

See page 12 for Equation (4.7)

where $T_g(Q)$ - ~~some~~ kind of a mean temperature for point Q. The physical idea of the approximation adopted here is as follows: the velocity of the particles leaving $d\sigma_Q$ is assumed to be constant, identical for all particles and equal to the mean velocity at a temperature $T_g(Q)$. With the aid of Laplace transform equation (4.7) is reduced to a corresponding equation for the stationary case.

$$n_r(P, \omega, t) = n_r^*(P, \omega, t) + \int\limits_{S_P} \frac{\omega_{PQ}\nu_P}{r_{PQ}^2} R(P, \omega_{PQ}, \omega) \times$$
$$\times n_r\left[Q, \omega_{PQ}, t - r_{PQ}\left(\frac{m}{4kT_g(Q)}\right)^{1/2}\right] d\sigma_Q \qquad (4.7)$$

We will discuss a case of small oscillations.We will do that with the aid of general equation (2.2). Assuming the values,figuring in this equation,are represented in form of

$$n(P, \omega, t) = n(P, \omega) + n'(P, \omega) e^{-ipt}$$
$$T_g(P, t) = T_g(P) + T_g'(P) e^{-ipt} \qquad (4.7a)$$

The values with primes appears to be complex here in conformity with the fact that theier phase, generally speaking, are variegated. Making a linearization of equation (2.2) and making, as previously, a substitution

$$\tau = r_{PQ}\left[\frac{m}{2kT_g(Q)}\right]^{1/2} u \qquad (4.7b)$$

it is possible to reduce same into form

See page 12a for Equation (4.8)

in which time figures no longer.The form of this equation allows to make the following important conclusions.In view of the presense of an exponential multiple the integral according to u is taken actually only within limits

$$\frac{1}{2} 2^{-1/2} \leqslant u \leqslant \frac{3}{2} 2^{-1/2} \qquad (4.8a)$$

At given frequency,consequently, the mutual influense is noticeable only for not too distant from each other parts of the surface

$$r_{PQ} \leqslant \frac{1}{p}\left(\frac{4kT(Q)}{m}\right)^{1/2} \qquad (4.8b)$$

because otherwise the function under the sign of the integral oscillates sharply. At higher frequencies,such, that

$$p \gg \frac{1}{r_{PQ}}\left(\frac{4kT(Q)}{m}\right)^{1/2} \qquad (4.8c)$$

for all existing a_{rPQ} repeated reflections generally produce no effect. The oscillation with low frequency can be treated as a quasistationary movement. Thus

See page 12a for Equation (4.8d)

$$n_r'(P,\omega) = n_r^{*\prime}(P,\omega) + \left(\frac{16}{\pi}\right)^{1/2} \int\limits_{S_P} \frac{\omega_{PQ}\nu_P}{r_{PQ}^2} R(P,\omega_{PQ},\omega)\, n_r(Q,\omega_{PQ}) \times$$
$$\times \int\limits_0^\infty \left[\frac{n_r'(Q,\omega_{PQ})}{n_r(Q,\omega_{PQ})} + \left(-\frac{3}{2} + \frac{1}{u^2}\right)\frac{T'(Q)}{T(Q)}\right] u^{-4} \times$$
$$\times \exp\left[-u^{-2} + ir_{PQ}\left(\frac{m}{2kT(Q)}\right)^{1/2} pu\right] du\, d\sigma_Q \qquad (4.8)$$

$$n_r'(P,\omega) = n_r^{*\prime}(P,\omega),\quad p \gg \frac{1}{r_{PQ}}\left(\frac{4kT(Q)}{m}\right)^{1/2}$$

$$n_r'(P,\omega) = n_r^{*\prime}(P,\omega) + \int\limits_{S_P} \frac{\omega_{PQ}\nu_P}{r_{PQ}^2} R(P,\omega_{PQ},\omega)\, n_r'(Q,\omega_{PQ})\, d\sigma_Q$$
$$p \ll \frac{1}{r_{PQ}}\left(\frac{4kT(Q)}{m}\right)^{1/2} \qquad (4.8d)$$

If it will become possible to solve (4.8) for any given frequency then by superimposing its solutions it is possible to describe any given small movement.

Submitted 9-22-1960

Literature

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